Package 'eba'

April 18, 2025

Version 1.10-1

Date 2025-04-18

Title Elimination-by-Aspects Models

Depends R (>= 4.0.0), stats, graphics

Imports nlme, psychotools

Description Fitting and testing multi-attribute probabilistic choice models, especially the Bradley-Terry-Luce (BTL) model (Bradley & Terry, 1952 <doi:10.1093/biomet/39.3-4.324>; Luce, 1959), elimination-by-aspects (EBA) models (Tversky, 1972 <doi:10.1037/h0032955>), and preference tree (Pretree) models (Tversky & Sattath, 1979 <doi:10.1037/0033-295X.86.6.542>).

License GPL (>= 2)

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NeedsCompilation no

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Repository CRAN

Date/Publication 2025-04-18 16:30:02 UTC

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balanced.pcdesign   *Balanced Paired-Comparison Design*

Description

Creates a (completely) balanced paired-comparison design.

Usage

```
balanced.pcdesign(nstimuli)
```

Arguments

nstimuli   number of stimuli in the paired-comparison design

Details

When `nstimuli` is odd, the presentation order is completely balanced, that is any given stimulus appears an equal number of times as the first and second member of a pair. When `nstimuli` is even, the presentation order is balanced as much as possible. Subjects should be equally assigned to `listA` and `listB` for the purpose of balancing the within-pair presentation order across a sample of subjects. Pairs should be re-randomized for each subject. See David (1988) for details.

Value

pairs	a character array holding the balanced pairs
listA	the vector pairs in the original within-pair order
listB	the vector of pairs in the inverted within-pair order

References

David, H. (1988). *The method of paired comparisons*. London: Griffin.

See Also

`pcX`, `eba`.

Examples

```
## Create balanced design for 6 stimuli
bp <- balanced.pcdesign(6)

## Replicate each within-pair order 10 times and re-randomize
cbind(replicate(10, sample(bp$listA)), replicate(10, sample(bp$listB)))
```

---

boot	*Bootstrap for Elimination-by-Aspects (EBA) Models*

Description

Performs a bootstrap by resampling the individual data matrices.

Usage

```
boot(D, R = 100, A = 1:I, s = rep(1/J, J), constrained = TRUE)
```

Arguments

D	either a 3d array consisting of the individual paired comparison matrices or an object of class `paircomp`
R	the number of bootstrap samples
A	a list of vectors consisting of the stimulus aspects; the default is 1:I, where I is the number of stimuli
s	the starting vector with default 1/J for all parameters, where J is the number of parameters
constrained	logical, if TRUE (default), parameters are constrained to be positive

Details

The bootstrap functions `eba.boot.constrained` and `eba.boot` are automatically called by `boot`.

The code is experimental and may change in the future.

Value

p	the matrix of bootstrap vectors
stat	the matrix of bootstrap statistics, including parameter means, standard errors, and confidence limits

See Also

eba, simulate.eba, paircomp.

Examples

```
data(pork)  # pork tasting data, 10 individual paired comparison matrices
m <- eba(apply(pork, 1:2, sum))  # fit Bradley-Terry-Luce model
b <- boot(pork, R = 200)         # resample 200 times

plot(coef(m), b$stat[, "mean"], log = "xy")
abline(0, 1, lty = 2)
```

---

celebrities	*Choice among Celebrities*

Description

Rumelhart and Greeno (1971) presented 234 participants with pairs of names of nine celebrities: the politicians L. B. Johnson (LBJ), Harold Wilson (HW), and Charles De Gaulle (CDG); the athletes Johnny Unitas (JU), Carl Yastrzemski (CY), and A. J. Foyt (AJF); the actresses Brigitte Bardot (BB), Elizabeth Taylor (ET), and Sophia Loren (SL). Participants were instructed to choose the person with whom they would rather spend an hour of discussion.

Usage

```
data(celebrities)
```

Format

A square data frame containing the absolute choice frequencies and a diagonal of zeros; row stimuli are chosen over column stimuli.

Source

Rumelhart, D.L., & Greeno, J.G. (1971). Similarity between stimuli: An experimental test of the Luce and Restle choice models. *Journal of Mathematical Psychology*, **8**, 370–381. doi:10.1016/00222496(71)900381

Examples

```
data(celebrities)
celebrities["LBJ", "HW"]  # 159 participants chose Johnson over Wilson
```

---

circular	*Circular Triads (Intransitive Cycles)*

Description

Number of circular triads and coefficient of consistency.

Usage

```
circular(mat, alternative = c("two.sided", "less", "greater"),
        exact = NULL, correct = TRUE, simulate.p.value = FALSE,
        nsim = 2000)
```

Arguments

mat	a square matrix or a data frame consisting of (individual) binary choice data; row stimuli are chosen over column stimuli
alternative	a character string specifying the alternative hypothesis, must be one of `"two.sided"` (default), `"less"` or `"greater"`
exact	a logical indicating whether an exact p-value should be computed
correct	a logical indicating whether to apply continuity correction in the chi-square approximation for the p-value
simulate.p.value	
	a logical indicating whether to compute p-values by Monte Carlo simulation
nsim	an integer specifying the number of replicates used in the Monte Carlo test

Details

Kendall's coefficient of consistency,

$$zeta = 1 - T/T_{max},$$

lies between one (perfect consistency) and zero, where T is the observed number of circular triads, and the maximum possible number of circular triads is $T_{max} = n(n^2 - 4)/24$, if n is even, and $T_{max} = n(n^2 - 1)/24$ else, and n is the number of stimuli or objects being judged. For details see Kendall and Babington Smith (1940) and David (1988).

Kendall (1962) discusses a test of the hypothesis that the number of circular triads T is different (smaller or greater) than expected when choosing randomly. For small n, an exact p-value is computed, based on the exact distributions listed in Alway (1962) and in Kendall (1962). Otherwise, an approximate chi-square test is computed. In this test, the sampling distribution is measured from lower to higher values of T, so that the probability that T will be exceeded is the complement of the probability for chi2. The chi-square approximation may be incorrect if $n < 8$ and is only available for $n > 4$.

Value

`T`	number of circular triads
`T.max`	maximum possible number of circular triads
`T.exp`	expected number of circular triads $E(T)$ when choices are totally random
`zeta`	Kendall's coefficient of consistency
`chi2, df, correct`	the chi-square statistic and degrees of freedom for the approximate test, and whether continuity correction has been applied
`p.value`	the p-value for the test (see Details)
`simulate.p.value, nsim`	whether the p-value is based on simulations, number of simulation runs

References

Alway, G.G. (1962). The distribution of the number of circular triads in paired comparisons. *Biometrika*, **49**, 265–269. doi:10.1093/biomet/49.12.265

David, H. (1988). *The method of paired comparisons*. London: Griffin.

Kendall, M.G. (1962). *Rank correlation methods*. London: Griffin.

Kendall, M.G., & Babington Smith, B. (1940). On the method of paired comparisons. *Biometrika*, **31**, 324–345. doi:10.1093/biomet/31.34.324

See Also

eba, strans, kendall.u.

Examples

```
# A dog's preferences for six samples of food
# (Kendall and Babington Smith, 1940, p. 326)
dog <- matrix(c(0, 1, 1, 0, 1, 1,
                0, 0, 0, 1, 1, 0,
                0, 1, 0, 1, 1, 1,
                1, 0, 0, 0, 0, 0,
                0, 0, 0, 1, 0, 1,
                0, 1, 0, 1, 0, 0), 6, 6, byrow=TRUE)
dimnames(dog) <- setNames(rep(list(c("meat", "biscuit", "chocolate",
                                     "apple", "pear", "cheese")), 2),
                     c(">", "<"))
circular(dog, alternative="less")  # moderate consistency
subset(strans(dog)$violdf, !wst)   # list circular triads
```

---

cov.u   *Covariance Matrix of the EBA Utility Scale*

Description

Computes the (normalized) covariance matrix of the utility scale from the covariance matrix of elimination-by-aspects (EBA) model parameters.

Usage

```
cov.u(object, norm = "sum")
```

Arguments

object   an object of class eba, typically the result of a call to eba

norm   either sum (default), a number from 1 to number of stimuli, or NULL; see uscale
               for details

Details

The additivity rule for covariances $cov(x + y, z) = cov(x, z) + cov(y, z)$ is used for the computations.

If norm is not NULL, the unnormalized covariance matrix is transformed using $a^2 cov(u)$, where the constant a results from the type of normalization applied.

Value

The (normalized) covariance matrix of the utility scale.

See Also

uscale, eba, wald.test.

---

drugrisk   *Perceived Health Risk of Drugs*

Description

In summer 2007, a survey was conducted at the Department of Psychology, University of Tuebingen. Hundred and ninety-two participants were presented with all 15 unordered pairs of the names of six drugs or substances and asked to choose the drug they judged as more dangerous for their health. The six drugs were alcohol (alc), tobacco (tob), cannabis (can), ecstasy (ecs), heroine (her), and cocaine (coc). Choice frequencies were aggregated in four groups defined by gender and age.

Usage

```
data(drugrisk)
```

Format

A 3d array consisting of four square matrices of choice frequencies (row drugs are judged over column drugs):

drugrisk[, , group = "female30"] holds the choices of the 48 female participants up to 30 years of age.

drugrisk[, , group = "female31"] holds the choices of the 48 female participants from 31 years of age.

drugrisk[, , group = "male30"] holds the choices of the 48 male participants up to 30 years of age.

drugrisk[, , group = "male31"] holds the choices of the 48 male participants from 31 years of age.

Source

Wickelmaier, F. (2008). Analyzing paired-comparison data in R using probabilistic choice models. Presented at the *R User Conference 2008*, August 12-14, Dortmund, Germany.

Examples

```
data(drugrisk)

## Bradley-Terry-Luce (BTL) model
btl <- eba(drugrisk[, , group = "male30"])

## Elimination-by-aspects (EBA) model, 1 additional aspect
A1 <- list(c(1), c(2,7), c(3,7), c(4,7), c(5,7), c (6,7))
eba1 <- eba(drugrisk[, , group = "male30"], A1)

## EBA model, 2 additional aspects
A2 <- list(c(1), c(2,7), c(3,7), c(4,7,8), c(5,7,8), c(6,7,8))
eba2 <- eba(drugrisk[, , group = "male30"], A2)

## Model selection
anova(btl, eba1, eba2)

## Utility scale values (BTL for females, EBA for males)
dotchart(cbind(
  apply(drugrisk[, , 1:2], 3, function(x) uscale(eba(x),     norm = 1)),
  apply(drugrisk[, , 3:4], 3, function(x) uscale(eba(x, A2), norm = 1))
  ), xlab="Utility scale value (BTL and EBA models)",
    main="Perceived health risk of drugs",
  panel.first = abline(v = 1, col = "gray"), log = "x")
mtext("(Wickelmaier, 2008)", line = 0.5)
```

---

eba   *Elimination-by-Aspects (EBA) Models*

Description

Fits a (multi-attribute) probabilistic choice model by maximum likelihood.

Usage

```
eba(M, A = 1:I, s = rep(1/J, J), constrained = TRUE)

OptiPt(M, A = 1:I, s = rep(1/J, J), constrained = TRUE)

## S3 method for class 'eba'
summary(object, ...)

## S3 method for class 'eba'
anova(object, ..., test = c("Chisq", "none"))
```

Arguments

M	a square matrix or a data frame consisting of absolute choice frequencies; row stimuli are chosen over column stimuli
A	a list of vectors consisting of the stimulus aspects; the default is 1:I, where I is the number of stimuli
s	the starting vector with default 1/J for all parameters, where J is the number of parameters
constrained	logical, if TRUE (default), parameters are constrained to be positive
object	an object of class eba, typically the result of a call to eba
test	should the p-values of the chi-square distributions be reported?
...	additional arguments; none are used in the summary method; in the anova method they refer to additional objects of class eba.

Details

eba is a wrapper function for OptiPt. Both functions can be used interchangeably. See Wickelmaier and Schmid (2004) for further details.

The probabilistic choice models that can be fitted to paired-comparison data are the Bradley-Terry-Luce (BTL) model (Bradley, 1984; Luce, 1959), preference tree (Pretree) models (Tversky and Sattath, 1979), and elimination-by-aspects (EBA) models (Tversky, 1972), the former being special cases of the latter.

A represents the family of aspect sets. It is usually a list of vectors, the first element of each being a number from 1 to I; additional elements specify the aspects shared by several stimuli. A must have as many elements as there are stimuli. When fitting a BTL model, A reduces to 1:I (the default), i.e. there is only one aspect per stimulus.

The maximum likelihood estimation of the parameters is carried out by nlm. The Hessian matrix, however, is approximated by nlme::fdHess. The likelihood functions L.constrained and L are called automatically.

See [group.test](#) for details on the likelihood ratio tests reported by summary.eba.

Value

coefficients	a vector of parameter estimates
estimate	same as coefficients
logL.eba	the log-likelihood of the fitted model
logL.sat	the log-likelihood of the saturated (binomial) model
goodness.of.fit	the goodness of fit statistic including the likelihood ratio fitted vs. saturated model (-2logL), the degrees of freedom, and the p-value of the corresponding chi-square distribution
u.scale	the unnormalized utility scale of the stimuli; each utility scale value is defined as the sum of aspect values (parameters) that characterize a given stimulus
hessian	the Hessian matrix of the likelihood function
cov.p	the covariance matrix of the model parameters
chi.alt	the Pearson chi-square goodness of fit statistic
fitted	the fitted paired-comparison matrix
y1	the data vector of the upper triangle matrix
y0	the data vector of the lower triangle matrix
n	the number of observations per pair (y1 + y0)
mu	the predicted choice probabilities for the upper triangle
nobs	the number of pairs

Author(s)

Florian Wickelmaier

References

Bradley, R.A. (1984). Paired comparisons: Some basic procedures and examples. In P.R. Krishnaiah & P.K. Sen (eds.), *Handbook of Statistics, Volume 4*. Amsterdam: Elsevier. doi:10.1016/S01697161(84)040165

Luce, R.D. (1959). *Individual choice behavior: A theoretical analysis*. New York: Wiley.

Tversky, A. (1972). Elimination by aspects: A theory of choice. *Psychological Review*, **79**, 281–299. doi:10.1037/h0032955

Tversky, A., & Sattath, S. (1979). Preference trees. *Psychological Review*, **86**, 542–573. doi:10.1037/0033295X.86.6.542

Wickelmaier, F., & Schmid, C. (2004). A Matlab function to estimate choice model parameters from paired-comparison data. *Behavior Research Methods, Instruments, and Computers*, **36**, 29–40. doi:10.3758/BF03195547

See Also

strans, uscale, cov.u, group.test, wald.test, plot.eba, residuals.eba, logLik.eba, simulate.eba, kendall.u, circular, trineq, thurstone, nlm.

Examples

```
data(celebrities)                 # absolute choice frequencies
btl1 <- eba(celebrities)          # fit Bradley-Terry-Luce model
A <- list(c(1,10), c(2,10), c(3,10),
          c(4,11), c(5,11), c(6,11),
          c(7,12), c(8,12), c(9,12))  # the structure of aspects
eba1 <- eba(celebrities, A)       # fit elimination-by-aspects model

summary(eba1)                     # goodness of fit
plot(eba1)                        # residuals versus predicted values
anova(btl1, eba1)                 # model comparison based on likelihoods
confint(eba1)                     # confidence intervals for parameters
uscale(eba1)                      # utility scale

ci <- 1.96 * sqrt(diag(cov.u(eba1)))     # 95% CI for utility scale values
dotchart(uscale(eba1), xlim=c(0, .3), main="Choice among celebrities",
         xlab="Utility scale value (EBA model)", pch=16)    # plot the scale
arrows(uscale(eba1)-ci, 1:9, uscale(eba1)+ci, 1:9, .05, 90, 3)  # error bars
abline(v=1/9, lty=2)                     # indifference line
mtext("(Rumelhart and Greeno, 1971)", line=.5)

## See data(package = "eba") for application examples.
```

---

eba.order   *Elimination-by-Aspects (EBA) Models with Order-Effect*

Description

Fits a (multi-attribute) probabilistic choice model that accounts for the effect of the presentation order within a pair.

Usage

```
eba.order(M1, M2 = NULL, A = 1:I, s = c(rep(1/J, J), 1),
          constrained = TRUE)

## S3 method for class 'eba.order'
summary(object, ...)
```

Arguments

M1, M2   two square matrices or data frames consisting of absolute choice frequencies in both within-pair orders; row stimuli are chosen over column stimuli. If M2 is empty (default), M1 is assumed to be a 3d array containing both orders

A	see [eba](eba)
s	the starting vector with default 1/J for all J aspect parameters, and 1 for the order effect
constrained	see [eba](eba)
object	an object of class eba.order, typically the result of a call to eba.order
...	additional arguments

Details

The choice models include a single multiplicative order effect, order, that is constant for all pairs (see Davidson and Beaver, 1977). An order effect < 1 (> 1) indicates a bias in favor of the first (second) interval. See [eba](eba) for choice models without order effect.

Several likelihood ratio tests are performed (see also summary.eba).

EBA.order tests an order-effect EBA model against a saturated binomial model; this corresponds to a goodness of fit test of the former model.

Order tests an EBA model with an order effect constrained to 1 against an unconstrained order-effect EBA model; this corresponds to a test of the order effect.

Effect tests an order-effect indifference model (where all scale values are equal, but the order effect is free) against the order-effect EBA model; this corresponds to testing for a stimulus effect; order0 is the estimate of the former model.

Wickelmaier and Choisel (2006) describe a model that generalizes the Davidson-Beaver model and allows for an order effect in Pretree and EBA models.

Value

coefficients	a vector of parameter estimates, the last component holds the order-effect estimate
estimate	same as coefficients
logL.eba	the log-likelihood of the fitted model
logL.sat	the log-likelihood of the saturated (binomial) model
goodness.of.fit	the goodness of fit statistic including the likelihood ratio fitted vs. saturated model (-2logL), the degrees of freedom, and the p-value of the corresponding chi-square distribution
u.scale	the unnormalized utility scale of the stimuli; each utility scale value is defined as the sum of aspect values (parameters) that characterize a given stimulus
hessian	the Hessian matrix of the likelihood function
cov.p	the covariance matrix of the model parameters
chi.alt	the Pearson chi-square goodness of fit statistic
fitted	3d array of the fitted paired-comparison matrices
y1	the data vector of the upper triangle matrices
y0	the data vector of the lower triangle matrices
n	the number of observations per pair (y1 + y0)
mu	the predicted choice probabilities for the upper triangles
M1, M2	the data matrices

Author(s)

Florian Wickelmaier

References

Davidson, R.R., & Beaver, R.J. (1977). On extending the Bradley-Terry model to incorporate within-pair order effects. *Biometrics*, **33**, 693–702.

Wickelmaier, F., & Choisel, S. (2006). Modeling within-pair order effects in paired-comparison judgments. In D.E. Kornbrot, R.M. Msetfi, & A.W. MacRae (eds.), *Fechner Day 2006. Proceedings of the 22nd Annual Meeting of the International Society for Psychophysics* (p. 89–94). St. Albans, UK: The ISP.

See Also

eba, group.test, plot.eba, residuals.eba, logLik.eba.

Examples

```
data(heaviness)                  # weights judging data
ebao1 <- eba.order(heaviness)    # Davidson-Beaver model
summary(ebao1)                   # goodness of fit
plot(ebao1)                      # residuals versus predicted values
confint(ebao1)                   # confidence intervals for parameters
```

---

envirosound	*Auditory Unpleasantness of Environmental Sound*

Description

Zimmer et al. (2004) investigated the auditory unpleasantness of twelve short binaural recordings (Johannsen and Prante, 2001); recordings were presented via headphones to 74 participants.

Usage

```
data(envirosound)
```

Format

A data frame containing 74 observations on 2 variables:

unpleasantness paired comparison of class paircomp; judgments for all 66 paired comparisons from 12 recordings: circular saw, stadium, dentist's drill, waterfall, ship's horn, stone in well, typewriter, hooves, fan, howling wind, tyre on gravel, wasp.

rt median response time.

Details

Details of the recordings, including psychoacoustic metrics, are available as an attribute of the unpleasantness variable (see Examples).

Source

Zimmer, K., Ellermeier, W., & Schmid, C. (2004). Using probabilistic choice models to investigate auditory unpleasantness. *Acta Acustica united with Acustica*, **90**(6), 1019–1028.

References

Johannsen, K., & Prante, H.U. (2001). Environmental sounds for psychoacoustic testing. *Acta Acustica united with Acustica*, **87**(2), 290–293.

See Also

eba, strans, paircomp.

Examples

```
requireNamespace("psychotools")
data(envirosound)
set.seed(1019)

## Choice-model representation of unpleasantness
mat <- summary(envirosound$unpleasantness, pcmatrix = TRUE)
strans(mat)
btl1 <- eba(mat)
eba1 <- eba(mat, A = list(c(1, 13), c(2, 13), c(3, 13), c(4, 13),
                          c(5, 13), c(6, 13), c(7, 13), c(8, 13),
                          c(9, 13), c(10, 13), c(11, 13), 12))
eba2 <- eba(mat, A = list(c(1, 13), c(2, 13), c(3, 13), c(4, 13),
                          c(5, 13), c(6, 13), c(7, 13, 14), c(8, 13, 14),
                          c(9, 13, 14), c(10, 13, 14), c(11, 13, 14), 12),
            s = runif(14))
anova(btl1, eba1, eba2)

sounds <- psychotools::covariates(envirosound$unpleasantness)
sounds$u <- 10 * uscale(eba2, norm = 9)  # u(fan) := 10

plot(magnitude ~ u, sounds, log = "x", type = "n",
     xlab = "Indirect scaling (EBA model)",
     ylab = "Direct magnitude estimation",
     main = "Auditory unpleasantness of environmental sound")
mtext("(Zimmer et al., 2004)", line = 0.5)
abline(lm(magnitude ~ log10(u), sounds))
text(magnitude ~ u, sounds, labels = abbreviate(rownames(sounds), 4))

## Predicting unpleasantness from psychoacoustic metrics
summary(
  lm(log(u) ~ scale(sharpness, scale = FALSE) +
              scale(roughness, scale = FALSE):I(loudness.5 > 27),
     sounds[-12, ])  # w/o wasp
)
```

---

group.test   *Group Effects in Elimination-by-Aspects (EBA) Models*

Description

Tests for group effects in elimination-by-aspects (EBA) models.

Usage

```
group.test(groups, A = 1:I, s = rep(1/J, J), constrained = TRUE)
```

Arguments

groups	a 3d array containing one aggregate choice matrix per group
A	a list of vectors consisting of the stimulus aspects; the default is 1:I, where I is the number of stimuli
s	the starting vector with default 1/J for all parameters, where J is the number of parameters
constrained	logical, if TRUE (default), EBA parameters are constrained to be positive

Details

The five tests are all based on likelihood ratios.

Overall compares a 1-parameter Poisson model to a saturated Poisson model, thereby testing the equality of the frequencies in each cell of the array. This test corresponds to simultaneously testing for a null effect of (1) the context induced by a given pair, (2) the grouping factor, (3) the stimuli, and (4) the imbalance between pairs. The deviances of the remaining tests sum to the total deviance associated with the overall test.

EBA.g tests an EBA group model against a saturated binomial group model, which corresponds to a goodness of fit test of the EBA group model.

Group tests an EBA model having its parameters restricted to be equal across groups (single set of parameters) against the EBA group model allowing its parameters to vary freely across groups (one set of parameters per group); this corresponds to testing for group differences.

Effect tests an indifference model (where all choice probabilities are equal to 0.5) against the restricted EBA model (single set of parameters), which corresponds to testing for a stimulus effect.

Imbalance tests for differences in the number of observations per pair by comparing the average sample size (1-parameter Poisson model) to the actual sample sizes (saturated Poisson model).

See Duineveld, Arents, and King (2000) for further details, and Choisel and Wickelmaier (2007) for an application.

Value

tests	a table displaying the likelihood ratio test statistics

References

Choisel, S., & Wickelmaier, F. (2007). Evaluation of multichannel reproduced sound: Scaling auditory attributes underlying listener preference. *Journal of the Acoustical Society of America*, **121**, 388–400. doi:10.1121/1.2385043

Duineveld, C.A.A., Arents, P., & King, B.M. (2000). Log-linear modelling of paired comparison data from consumer tests. *Food Quality and Preference*, **11**, 63–70. doi:10.1016/s0950-3293(99)000403

See Also

eba, wald.test.

Examples

```
## Bradley-Terry-Luce model
data(pork)          # Is there a difference between Judge 1 and Judge 2?
groups <- simplify2array(list(apply(pork[, , 1:5],  1:2, sum),
                              apply(pork[, , 6:10], 1:2, sum)))
group.test(groups)  # Yes, there is.

## Elimination-by-aspects model
data(drugrisk)   # Do younger and older males judge risk of drugs differently?
A2 <- list(c(1), c(2,7), c(3,7), c(4,7,8), c(5,7,8), c(6,7,8))
group.test(drugrisk[, , 3:4], A2)  # Yes.
```

---

heaviness	*Weights Judging Data*

Description

Beaver and Gokhale (1975) presented fifty subjects with all 20 ordered pairs of bottles filled with lead shot and asked them to choose the bottle that felt heavier. The mass of the bottles was 90, 95, 100, 105, and 110 grams, respectively. Choice frequencies were aggregated across subjects for the two within-pair presentation orders.

Usage

```
data(heaviness)
```

Format

A 3d array consisting of two square matrices:

heaviness[, , order = 1] holds the choices where the row stimulus was presented first for each pair (in the upper triangle, and vice versa in the lower triangle).

heaviness[, , order = 2] holds the choices where the column stimulus was presented first for each pair (in the upper triangle, and vice versa in the lower triangle).

Source

Beaver, R.J., & Gokhale, D.V. (1975). A model to incorporate within-pair order effects in paired comparisons. *Communications in Statistics*, **4**, 923–939. doi:10.1080/03610927308827302

See Also

`eba.order` for a model that includes a within-pair order effect.

Examples

```
data(heaviness)
## 6 subjects chose 90g over 100g, when 90g was presented first.
heaviness["90g", "100g", order=1]

## 44 subjects chose 100g over 90g, when 90g was presented first.
heaviness["100g", "90g", order=1]

## 14 subjects chose 90g over 100g, when 90g was presented second.
heaviness["90g", "100g", order=2]

## 36 subjects chose 100g over 90g, when 90g was presented second.
heaviness["100g", "90g", order=2]

## Bradley-Terry-Luce (BTL) model for each within-pair order
btl1 <- eba(heaviness[, , 1])
btl2 <- eba(heaviness[, , 2])

xval <- seq(90, 110, 5)
plot(uscale(btl1) ~ xval, type="n", log="y",
     xlab="Mass of lead shot filled bottle (in g)",
     ylab="Perceived heaviness (BTL model)",
     main="Weights judging experiment")
mtext("(Beaver and Gokhale, 1975)", line=.5)

arrows(xval, uscale(btl1) - 1.96*sqrt(diag(cov.u(btl1))),
       xval, uscale(btl1) + 1.96*sqrt(diag(cov.u(btl1))), .05, 90, 3, "gray")
arrows(xval, uscale(btl2) - 1.96*sqrt(diag(cov.u(btl2))),
       xval, uscale(btl2) + 1.96*sqrt(diag(cov.u(btl2))), .05, 90, 3, "gray")

points(uscale(btl1) ~ xval, type="b", pch=16)
points(uscale(btl2) ~ xval, type="b", lty=2, pch=21, bg="white")

text(90.3, .01,  "Lower weight judged first", adj=0)
text(90.3, .08, "Higher weight judged first", adj=0)
```

---

inclusion.rule　　　　　*Inclusion Rule*

Description

Checks if a family of sets fulfills the inclusion rule.

Usage

```
inclusion.rule(A)
```

Arguments

A   a list of vectors consisting of the stimulus aspects of an elimination-by-aspects model

Details

The inclusion rule is necessary and sufficient for a tree structure on the aspect sets:

Structure theorem. A family $\{x'|x \in T\}$ of aspect sets is representable by a tree iff either $x' \cap y' \supset x' \cap z'$ or $x' \cap z' \supset x' \cap y'$ for all x, y, z in T. (Tversky and Sattath, 1979, p. 546)

Value

Either TRUE if the inclusion rule holds for A, or FALSE otherwise.

References

Tversky, A., & Sattath, S. (1979). Preference trees. *Psychological Review*, **86**, 542–573. doi:10.1037/0033295X.86.6.542

See Also

eba, trineq, strans.

Examples

```
A <- list(c(1, 5), c(2, 5), c(3, 6), c(4, 6))  # tree
inclusion.rule(A)

B <- list(c(1, 5), c(2, 5, 6), c(3, 6), c(4, 6))  # lattice
inclusion.rule(B)
```

---

kendall.u *Kendall's Coefficient of Agreement*

Description

Kendall's u coefficient of agreement between judges.

Usage

```
kendall.u(M, correct = TRUE)
```

Arguments

M	a square matrix or a data frame consisting of absolute choice frequencies; row stimuli are chosen over column stimuli
correct	logical, if TRUE (default) a continuity correction is applied when computing the test statistic (by subtracting one from the sum of agreeing pairs)

Details

Kendall's u (Kendall and Babington Smith, 1940) takes on values between min.u (minimum agreement) and 1 (maximum agreement). The minimum min.u equals $-1/(m-1)$, if m is even, and $-1/m$, if m is odd, where m is the number of subjects (judges).

The null hypothesis in the chi-square test is that the agreement between judges is by chance.

It is assumed that there is an equal number of observations per pair and that each subject judges each pair only once.

Value

u	Kendall's u coefficient of agreement
min.u	the minimum value for u
chi2	the chi-square statistic for a test that the agreement is by chance
df	the degrees of freedom
p.value	the p-value of the test

References

Kendall, M.G., & Babington Smith, B. (1940). On the method of paired comparisons. *Biometrika*, **31**, 324–345. doi:10.1093/biomet/31.34.324

See Also

schoolsubjects, eba, strans, circular.

Examples

```
data(schoolsubjects)
lapply(schoolsubjects, kendall.u)  # better-than-chance agreement
```

---

linear2btl   *Linear Coefficients to Bradley-Terry-Luce (BTL) Estimates*

Description

Transforms linear model coefficients to Bradley-Terry-Luce (BTL) model parameter estimates.

Usage

```
linear2btl(object, order = FALSE)
```

Arguments

object   an object of class `glm` or `lm` specifying a BTL model

order   logical, does the model include an order effect? Defaults to FALSE

Details

The design matrix used by `glm` or `lm` usually results from a call to [pcX](#). It is assumed that the reference category is the first level. The covariance matrix is estimated by employing the delta method. See Imrey, Johnson, and Koch (1976) for more details.

Value

btl.parameters a matrix; the first column holds the BTL parameter estimates, the second column the approximate standard errors

cova   the approximate covariance matrix of the BTL parameter estimates

linear.coefs a vector of the original linear coefficients as returned by `glm` or `lm`

References

Imrey, P.B., Johnson, W.D., & Koch, G.G. (1976). An incomplete contingency table approach to paired-comparison experiments. *Journal of the American Statistical Association*, **71**, 614–623. doi:10.2307/2285591

See Also

eba, eba.order, glm, pcX.

Examples

```
data(drugrisk)
y1 <- t(drugrisk[, , 1])[lower.tri(drugrisk[, , 1])]
y0 <-   drugrisk[, , 1][ lower.tri(drugrisk[, , 1])]

## Fit BTL model using glm (maximum likelihood)
btl.glm <- glm(cbind(y1, y0) ~ 0 + pcX(6), binomial)
linear2btl(btl.glm)

## Fit BTL model using lm (weighted least squares)
btl.lm <- lm(log(y1/y0) ~ 0 + pcX(6), weights=y1*y0/(y1 + y0))
linear2btl(btl.lm)
```

---

logLik.eba   *Log-Likelihood of an eba Object*

Description

Returns the log-likelihood value of the (multi-attribute) probabilistic choice model represented by `object` evaluated at the estimated parameters.

Usage

```
## S3 method for class 'eba'
logLik(object, ...)
```

Arguments

object	an object inheriting from class eba, representing a fitted elimination-by-aspects model
...	some methods for this generic require additional arguments; none are used in this method.

Value

The log-likelihood of the model represented by `object` evaluated at the estimated parameters.

See Also

eba, logLik.lm, AIC.

Examples

```
data(heaviness)
btl1 <- eba(heaviness[, , order=1])
logLik(btl1)
AIC(btl1)
BIC(btl1)
```

---

mbt   *Mallows-Bradley-Terry Model*

Description

Fits a Mallows-Bradley-Terry (MBT) model by maximum likelihood.

Usage

```
mbt(data, bootstrap = FALSE, nsim = 1000, ...)
```

Arguments

data	a data frame, the first t columns containing the ranks, the (t + 1)th column containing the frequencies
bootstrap	logical. Return a parametric bootstrap p-value?
nsim	number of bootstrap replicates
...	further aguments passed to `simulate`

Details

`mbt` provides a front end for `glm`. See Critchlow and Fligner (1991) and Mallows (1957) for details.

Value

coefficients	a vector of parameter estimates (scale values) constrained to sum to unity
goodness.of.fit	
	the goodness of fit statistic including the likelihood ratio fitted vs. saturated model (-2logL), the degrees of freedom, the p-value of the corresponding chi-square distribution, and if `bootstrap` is `TRUE` the bootstrap p-value
perm.idx	the names of the non-zero frequency ranks
y	the vector of rank frequencies including zeros
mbt.glm	the output from a call to `glm`

Author(s)

Florian Wickelmaier

References

Critchlow, D.E., & Fligner, M.A. (1991). Paired comparison, triple comparison, and ranking experiments as generalized linear models, and their implementation in GLIM. *Psychometrika*, **56**, 517–533. doi:10.1007/bf02294488

Mallows, C.L. (1957). Non-null ranking models. I. *Biometrika*, **44**, 114–130. doi:10.1093/biomet/44.12.114

See Also

tartness, glm.

Examples

```
data(tartness)       # tartness rankings of salad dressings (Vargo, 1989)
mbt(tartness, bootstrap=TRUE, nsim=500)  # fit Mallows-Bradley-Terry model
```

---

pcX   *Paired-Comparison Design Matrix*

Description

Computes a paired-comparison design matrix.

Usage

```
pcX(nstimuli, omitRef = TRUE)
```

Arguments

nstimuli   number of stimuli in the paired-comparison design

omitRef   logical, if TRUE (default), the first column corresponding to the reference cate-
                gory is omitted

Details

The design matrix can be used when fitting a Bradley-Terry-Luce (BTL) model or a Thurstone-
Mosteller (TM) model by means of glm or lm. See Critchlow and Fligner (1991) for more details.

Value

A matrix having (nstimuli - 1)*nstimuli/2 rows and nstimuli - 1 columns (if the reference
category is omitted).

References

Critchlow, D.E., & Fligner, M.A. (1991). Paired comparison, triple comparison, and ranking ex-
periments as generalized linear models, and their implementation in GLIM. *Psychometrika*, **56**,
517–533. doi:10.1007/bf02294488

See Also

eba, thurstone, glm, balanced.pcdesign, linear2btl.

Examples

```
data(drugrisk)            # absolute choice frequencies
btl <- eba(drugrisk[, , 1])  # fit Bradley-Terry-Luce model using eba
summary(btl)

y1 <- t(drugrisk[, , 1])[lower.tri(drugrisk[, , 1])]
y0 <-   drugrisk[, , 1][ lower.tri(drugrisk[, , 1])]

## Fit Bradley-Terry-Luce model using glm
btl.glm <- glm(cbind(y1, y0) ~ 0 + pcX(6), binomial)
summary(btl.glm)

## Fit Thurstone Case V model using glm
tm.glm <- glm(cbind(y1, y0) ~ 0 + pcX(6), binomial(probit))
summary(tm.glm)
```

---

plot.eba   *Diagnostic Plot for EBA Models*

Description

Plots elimination-by-aspects (EBA) model residuals against fitted values.

Usage

```
## S3 method for class 'eba'
plot(x, xlab = "Predicted choice probabilities",
     ylab = "Deviance residuals", ...)
```

Arguments

x   an object of class eba, typically the result of a call to eba

xlab, ylab, ... graphical parameters passed to plot.

Details

The deviance residuals are plotted against the predicted choice probabilities for the upper triangle
of the paired-comparison matrix.

See Also

eba, residuals.eba.

Examples

```
## Compare two choice models

data(celebrities)                    # absolute choice frequencies
btl1 <- eba(celebrities)             # fit Bradley-Terry-Luce model
A <- list(c(1,10), c(2,10), c(3,10),
          c(4,11), c(5,11), c(6,11),
          c(7,12), c(8,12), c(9,12)) # the structure of aspects
eba1 <- eba(celebrities, A)          # fit elimination-by-aspects model
anova(btl1, eba1)                    # model comparison based on likelihoods

par(mfrow = 1:2)                     # residuals versus fitted values
plot(btl1, main = "BTL", ylim = c(-4, 4.5))  # BTL doesn't fit well
plot(eba1, main = "EBA", ylim = c(-4, 4.5))  # EBA fits better
```

---

pork   *Pork Tasting Data*

Description

Bradley and Terry (1952) provide the individual choice matrices of two judges choosing between pairs of three samples of pork meet. The pigs had been fed on either corn (C), corn plus peanut supplement (Cp), or corn plus a large peanut supplement (CP). Each judge does five repetitions.

Usage

```
data(pork)
```

Format

A 3d array consisting of ten square matrices. The first five matrices correspond to the five repetitions of the first judge, the last five to the repetitions of the second judge. Row stimuli are chosen (preferred) over column stimuli.

Source

Bradley, R.A., & Terry, M.E. (1952). Rank analysis of incomplete block designs. I. The method of paired comparisons. *Biometrika*, **39**, 324–345. doi:10.1093/biomet/39.34.324

Examples

```
data(pork)
apply(pork, 1:2, sum)  # aggregate choice frequencies
```

---

residuals.eba   *Residuals for EBA Models*

Description

Computes deviance and Pearson residuals for eba objects.

Usage

```
## S3 method for class 'eba'
residuals(object, type = c("deviance", "pearson"), ...)
```

Arguments

object	an object of class eba, typically the result of a call to eba
type	the type of residuals which should be returned; the alternatives are: "deviance" (default) and "pearson"
...	further arguments passed to or from other methods; none are used in this method.

Details

Residuals are computed from the upper triangle of the paired-comparison matrix. See residuals.glm for details.

Value

A vector of residuals having as many elements as pairs of stimuli.

See Also

eba, residuals.glm, plot.eba.

Examples

```
data(celebrities)              # absolute choice frequencies
btl1 <- eba(celebrities)       # fit Bradley-Terry-Luce model
sum( resid(btl1)^2 )           # deviance G2
deviance(btl1)
sum( resid(btl1, "pearson")^2 ) # Pearson X2
```

---

schoolsubjects   *Preference for School Subjects*

Description

Two classes of children (ages 11 to 13) were asked to state their preferences with respect to certain school subjects. Each child was given a sheet on which were written the possible pairs of subjects and asked to underline the one preferred in each case (Kendall and Babington Smith, 1940).

Usage

```
data(schoolsubjects)
```

Format

A list containing two square matrices of aggregate choice frequencies (row entries are preferred over column entries):

schoolsubjects[["boys"]] holds the frequencies of 21 boys choosing among 13 school subjects: woodwork, gymnastics, art, science, history, geography, arithmetic, religion, English literature, commercial subjects, algebra, English grammar, geometry.

schoolsubjects[["girls"]] holds the frequencies of 25 girls choosing among 11 school subjects: gymnastics, science, art, domestic science, history, arithmetic, geography, English literature, religion, algebra, English grammar.

Source

Kendall, M.G., & Babington Smith, B. (1940). On the method of paired comparisons. *Biometrika*, **31**, 324–345. doi:10.1093/biomet/31.34.324

Examples

```
data(schoolsubjects)

m <- lapply(schoolsubjects, eba)  # Bradley-Terry-Luce (BTL) model

par(mfrow = 1:2)
dotchart(uscale(m$boys), main = "Boys' preferences", log = "x")
dotchart(uscale(m$girls), main = "Girls' preferences", log = "x")
mtext("Utility scale value (BTL model)", outer = TRUE, side = 1,
      line = -2)
mtext("(Kendall and Babington Smith, 1940)", outer = TRUE, line = -4)
```

---

simulate.eba   *Simulate Responses from EBA Models*

Description

Simulates responses from the distribution corresponding to a fitted eba model object.

Usage

```
## S3 method for class 'eba'
simulate(object, nsim, seed, pool = TRUE, ...)
```

Arguments

object	an object of class eba, typically the result of a call to eba
nsim	currently not used
seed	currently not used
pool	logical, if TRUE (default), pooled responses (summed across respondents) are returned
...	further arguments passed to or from other methods; none are used in this method

Details

Responses are simulated by rbinom with sizes taken from the original sample and probabilities computed from the model object.

Value

A paired-comparison matrix of (pooled) responses.

See Also

eba, boot.

Examples

```
data(celebrities)                     # absolute choice frequencies
A <- list(c(1,10), c(2,10), c(3,10),
          c(4,11), c(5,11), c(6,11),
          c(7,12), c(8,12), c(9,12))  # the structure of aspects
eba1 <- eba(celebrities, A)           # fit elimination-by-aspects model

## Parametric bootstrap of goodness-of-fit test
LR.stat <- replicate(200, deviance(eba(simulate(eba1), A)))

hist(LR.stat, col="lightgray", border="white", freq=FALSE, breaks=20,
     xlim=c(0, 60), main="Parametric bootstrap")
curve(dchisq(x, df=eba1$goodness.of.fit["df"]), add=TRUE)
abline(v=deviance(eba1), lty=2)
```

soundquality	*Quality of Multichannel Reproduced Sound*

Description

Paired comparison judgments of 40 selected listeners with respect to eight audio reproduction modes and four types of music: SQpreference includes judgments on overall preference; SQattributes includes judgments on specific spatial and timbral auditory attributes; SQsubjects: includes information about the listeners involved.

Usage

```
data("soundquality")
```

Format

SQpreference A data frame containing 783 observations on 6 variables:

id factor, listener ID.

time factor, listening experiment before or after elicitation and scaling of more specific auditory attributes.

progmat factor, the program material: Beethoven, Rachmaninov, Steely Dan, Sting.

repet the repetition number.

session the experimental session coding the presentation order of the program material.

preference paired comparison of class [paircomp](); preferences for all 28 paired comparisons from 8 audio reproduction modes: Mono, Phantom Mono, Stereo, Wide-Angle Stereo, 4-channel Matrix, 5-channel Upmix 1, 5-channel Upmix 2, and 5-channel Original.

SQattributes A data frame containing 156 observations on 10 variables:

id factor, listener ID.

progmat factor, the program material.

width, spaciousness, envelopment, distance, clarity, brightness, elevation, naturalness Paired comparison of class [paircomp]().

SQsubjects A data frame containing 78 observations on 18 variables:

id factor, listener ID

status factor, selection status; 40 listeners were selected.

HLmax maximum hearing level between 250 and 8000 Hz

stereowidth stereo-width discrimination threshold

fluency word fluency score

age subject age

gender factor, subject gender

education factor, education class

background, experience, listenmusic, concerts, instrument, critical, cinema, hifi, surround, earliertests indicators of prior experience

Details

The data were collected within a series of experiments conducted at the Sound Quality Research Unit (SQRU), Department of Acoustics, Aalborg University, Denmark, between September 2004 and March 2005.

The results of scaling listener preference and spatial and timbral auditory attributes are reported in Choisel and Wickelmaier (2007). See examples for replication code. Details about the loudspeaker setup and calibration are given in Choisel and Wickelmaier (2006). The attribute elicitation procedure is described in Wickelmaier and Ellermeier (2007) and in Choisel and Wickelmaier (2006). The selection of listeners for the experiments is described in Wickelmaier and Choisel (2005).

Portions of these data are also available via data("SoundQuality", package = "psychotools").

Note

One listener (ID 62) dropped out after contributing the first set of preference judgments.

References

Choisel, S., & Wickelmaier, F. (2006). Extraction of auditory features and elicitation of attributes for the assessment of multichannel reproduced sound. *Journal of the Audio Engineering Society*, **54**(9), 815–826.

Choisel, S., & Wickelmaier, F. (2007). Evaluation of multichannel reproduced sound: Scaling auditory attributes underlying listener preference. *Journal of the Acoustical Society of America*, **121**(1), 388–400. doi:10.1121/1.2385043

Wickelmaier, F., & Choisel, S. (2005). Selecting participants for listening tests of multichannel reproduced sound. Presented at the AES 118th Convention, May 28–31, Barcelona, Spain, convention paper 6483.

Wickelmaier, F., & Ellermeier, W. (2007). Deriving auditory features from triadic comparisons. *Perception & Psychophysics*, **69**(2), 287–297. doi:10.3758/BF03193750

See Also

eba, strans, paircomp.

Examples

```
requireNamespace("psychotools")
data(soundquality)

######### Replication code for Choisel and Wickelmaier (2007) ######

### A. Scaling overall preference

## Participants
summary(subset(SQsubjects, status == "selected"))

## Transitivity violations
aggregate(preference ~ progmat + time,
  data = SQpreference,
```

```
      function(x) unlist(strans(summary(x, pcmatrix = TRUE))[
                          c("weak", "moderate", "strong")]))

## BTL modeling
prefdf <- aggregate(preference ~ progmat + time,
             data = SQpreference,
             function(x) uscale(eba(summary(x, pcmatrix = TRUE))))

## Preference scale
p <- t(prefdf[prefdf$time == "before", 3])
colnames(p) <- levels(prefdf$progmat)
dotchart(p, main = "Quality of multichannel reproduced sound",
         xlab = "Estimated preference (BTL model)", log = "x",
         panel.first = abline(v = 1/8, col = "gray"))
points(x = t(prefdf[prefdf$time == "after", 3]),
       y = c(31:38, 21:28, 11:18, 1:8), pch = 3)
legend("topleft", c("before", "after"), pch = c(1, 3))
mtext("(Choisel and Wickelmaier, 2007)", line = 0.5)

### B. Scaling specific auditory attributes

## Transitivity violations
aggregate(
  x = SQattributes[-(1:2)],
  by = list(progmat = SQattributes$progmat),
  FUN = function(x) strans(summary(x, pcmatrix = TRUE))[
          c("weak", "moderate", "strong")],
  simplify = FALSE
)

## BTL modeling
uscal <- aggregate(
  x = SQattributes[-(1:2)],
  by = list(progmat = SQattributes$progmat),
  FUN = function(x) uscale(eba(summary(x, pcmatrix = TRUE)))
)
uscal <- data.frame(
  progmat = rep(levels(SQattributes$progmat), each = 8),
  repmod = factor(1:8, labels = labels(SQattributes$width)),
  as.data.frame(sapply(uscal[-1], t))
)

## EBA modeling: envelopment, width
uscal$envelopment[uscal$progmat == "SteelyDan"] <-
  uscale(eba(summary(SQattributes[SQattributes$progmat == "SteelyDan",
                                  "envelopment"], pcmatrix = TRUE),
          A = list(c(1,9), c(2,9), c(3,9), c(4,9), 5, 6, c(7,9), 8)))
uscal$width[uscal$progmat == "SteelyDan"] <-
  uscale(eba(summary(SQattributes[SQattributes$progmat == "SteelyDan",
                                  "width"], pcmatrix = TRUE),
          A = list(1, 2, c(3,9), c(4,9), c(5,9), 6, 7, c(8,9))))

### C. Relating overall preference to specific attributes
```

```r
## Principal components: classical music
pca1 <- princomp(
  ~ width + spaciousness + envelopment + distance +
    clarity + brightness + elevation,
  data = uscal,
  subset = progmat %in% c("Beethoven", "Rachmaninov"),
  cor = TRUE
)

## Loadings on first two components
L <- varimax(loadings(pca1) %*% diag(pca1$sdev)[, 1:2])

## Factor scores
f <- scale(predict(pca1)[, 1:2]) %*% L$rotmat
dimnames(f) <- list(
  abbreviate(rep(labels(SQattributes$width), 2), 2),
  names(pca1$sdev)[1:2]
)
biplot(f, 2*L$loadings, cex = 0.8, expand = 1.5,
       main = "Preference and auditory attributes: classical music",
       xlim = c(-1.5, 2), ylim = c(-2, 2))

## Predicting preference
classdf <- cbind(
  pref = as.vector(t(prefdf[prefdf$time == "after" &
          prefdf$progmat %in% c("Beethoven", "Rachmaninov"), 3])),
  as.data.frame(f)
)
m1 <- lm(pref ~ Comp.1 + Comp.2 + I(Comp.1^2), classdf)
c1 <- seq(-1.5, 2.0, length.out = 101)
c2 <- seq(-2.0, 2.0, length.out = 101)
z <- matrix(predict(m1, expand.grid(Comp.1 = c1, Comp.2 = c2)), 101, 101)
contour(c1, c2, z, add = TRUE, col = "darkblue")

## Principal components: pop music
pca2 <- princomp(
  ~ width + spaciousness + envelopment + distance +
    clarity + brightness + elevation,
  data = uscal,
  subset = progmat %in% c("SteelyDan", "Sting"),
  cor = TRUE
)
L <- varimax(loadings(pca2) %*% diag(pca2$sdev)[, 1:2])
f[] <- scale(predict(pca2)[, 1:2]) %*% L$rotmat
biplot(f, 2*L$loadings, cex = 0.8,
       main = "Preference and auditory attributes: pop music",
       xlim = c(-1.5, 2), ylim = c(-2.5, 1.5))

popdf <- cbind(
  pref = as.vector(t(prefdf[prefdf$time == "after" &
          prefdf$progmat %in% c("SteelyDan", "Sting"), 3])),
  as.data.frame(f)
```

```
)
m2 <- lm(pref ~ Comp.1 + Comp.2 + I(Comp.2^2), popdf)
c1 <- seq(-1.5, 2.0, length.out = 101)
c2 <- seq(-2.5, 1.5, length.out = 101)
z <- matrix(predict(m2, expand.grid(Comp.1 = c1, Comp.2 = c2)), 101, 101)
contour(c1, c2, z, add = TRUE, col = "darkblue")
```

---

strans	*Stochastic Transitivity*

Description

Checks the weak, moderate, and strong stochastic transitivity.

Usage

```
strans(M)
```

Arguments

M   a square matrix or a data frame consisting of absolute choice frequencies; row stimuli are chosen over column stimuli

Details

Weak (WST), moderate (MST), and strong (SST) stochastic transitivity hold for a set of choice probabilities P, whenever if $P_{ij} \geq 0.5$ and $P_{jk} \geq 0.5$, then

$P_{ik} \geq 0.5$ (WST),

$P_{ik} \geq \min(P_{ij}, P_{jk})$ (MST),

$P_{ik} \geq \max(P_{ij}, P_{jk})$ (SST).

See Suppes, Krantz, Luce, and Tversky (1989/2007, chap. 17) for an introduction to the representation of choice probabilities.

If WST holds, a permutation of the indices of the matrix exists such that the proportions in the upper triangular matrix are ≥ 0.5. This rearranged matrix is stored in pcm. If WST does not hold, cells in the upper triangular matrix that are smaller than 0.5 are replaced by 0.5. The deviance resulting from this restriction is reported in wst.fit.

The approximate likelihood ratio test for significance of the WST violations is according to Tversky (1969); for a more exact test of WST see Iverson and Falmagne (1985).

Value

A table displaying the number of violations of the weak, moderate, and strong stochastic transitivity, the number of tests, the error ratio (violations/tests), and the mean and maximum deviation from the minimum probability for which the corresponding transitivity would hold.

weak   number of violations of WST

moderate	number of violations of MST
strong	number of violations of SST
n.tests	number of transitivity tests performed
wst.violations	a vector containing $0.5 - P_{ik}$ for all triples that violate WST
mst.violations	a vector containing $\min(P_{ij}, P_{jk}) - P_{ik}$ for all triples that violate MST
sst.violations	a vector containing $\max(P_{ij}, P_{jk}) - P_{ik}$ for all triples that violate SST
pcm	the permuted square matrix of relative choice frequencies
ranking	the ranking of the objects, which corresponds to the colnames of pcm
chkdf	data frame reporting the choice proportions for each triple in each permutation
violdf	data frame reporting for each triple which type of transitivity holds or does not hold
wst.fit	likelihood ratio test of WST (see Details)
wst.mat	restricted matrix that satisfies WST

References

Iverson, G., & Falmagne, J.-C. (1985). Statistical issues in measurement. *Mathematical Social Sciences*, **10**, 131–153. doi:10.1016/01654896(85)900319

Suppes, P., Krantz, D.H., Luce, R.D., & Tversky, A. (1989/2007). *Foundations of measurement. Volume II*. Mineola, N.Y.: Dover Publications.

Tversky, A. (1969). Intransitivity of preferences. *Psychological Review*, **76**, 31–48. doi:10.1037/h0026750

See Also

eba, circular, kendall.u, trineq.

Examples

```
data(celebrities)              # absolute choice frequencies
strans(celebrities)            # WST and MST hold, but not SST
strans(celebrities)$pcm        # reordered relative frequencies
strans(celebrities)$violdf     # transitivity violations
```

| tartness | *Tartness Rankings of Salad Dressings* |

Description

The data were collected by Vargo (1989; cited in Critchlow and Fligner, 1991). Each of 32 judges is asked to rank four salad dressing preparations according to tartness, with a rank of 1 being assigned to the formulation judged to be the most tart.

Usage

```
data(tartness)
```

Format

A data frame consisting the rankings and their frequencies.

Source

Critchlow, D.E., & Fligner, M.A. (1991). Paired comparison, triple comparison, and ranking experiments as generalized linear models, and their implementation in GLIM. *Psychometrika*, **56**, 517–533. doi:10.1007/bf02294488

References

Vargo, M.D. (1989). Microbiological spoilage of a moderate acid food system using a dairy-based salad dressing model. Unpublished masters thesis, Ohio State University, Department of Food Science and Nutrition, Columbus, OH.

Examples

```
data(tartness)
```

---

thurstone	*Thurstone-Mosteller Model (Case V)*

Description

Fits a Thurstone-Mosteller model (Case V) by maximum likelihood.

Usage

```
thurstone(M)
```

Arguments

M   a square matrix or a data frame consisting of absolute choice frequencies; row stimuli are chosen over column stimuli

Details

thurstone provides a front end for glm. See Critchlow and Fligner (1991) for more details.

Value

estimate   a vector of parameter estimates (scale values), first element is set to zero

goodness.of.fit

the goodness of fit statistic including the likelihood ratio fitted vs. saturated model (-2logL), the degrees of freedom, and the p-value of the corresponding chi-square distribution

tm.glm   the output from a call to glm

References

Critchlow, D.E., & Fligner, M.A. (1991). Paired comparison, triple comparison, and ranking experiments as generalized linear models, and their implementation in GLIM. *Psychometrika*, **56**, 517–533. doi:10.1007/bf02294488

See Also

eba, strans, pcX, kendall.u, circular, glm.

Examples

```
## Taste data (David, 1988, p. 116)
taste <- matrix(c( 0,  3,  2,  2,
                  12,  0, 11,  3,
                  13,  4,  0,  5,
                  13, 12, 10,  0), 4, 4, byrow=TRUE)
dimnames(taste) <- setNames(rep(list(c("A1", "A2", "A3", "A4")), 2),
                            c(">", "<"))
thurstone(taste)  # Thurstone-Mosteller model fits OK
```

---

trineq   *Trinary Inequality*

Description

Checks if binary choice probabilities fulfill the trinary inequality.

Usage

```
trineq(M, A = 1:I)
```

Arguments

M   a square matrix or a data frame consisting of absolute choice frequencies; row stimuli are chosen over column stimuli

A   a list of vectors consisting of the stimulus aspects; the default is 1:I, where I is the number of stimuli

Details

For any triple of stimuli x, y, z, the trinary inequality states that, if $P(x, y) > 1/2$ and $(xy)z$, then

$$R(x, y, z) > 1,$$

where $R(x, y, z) = R(x, y)R(y, z)R(z, x)$, $R(x, y) = P(x, y)/P(y, x)$, and $(xy)z$ denotes that x and y share at least one aspect that z does not have (Tversky and Sattath, 1979, p. 554).

`inclusion.rule` checks if a family of aspect sets is representable by a tree.

Value

Results checking the trinary inequality.

n	number of tests of the trinary inequality
prop	proportion of triples confirming the trinary inequality
quant	quantiles of $R(x, y, z)$
n.tests	number of transitivity tests performed
chkdf	data frame reporting $R(x, y, z)$ for each triple where $P(x, y) > 1/2$ and $(xy)z$

References

Tversky, A., & Sattath, S. (1979). Preference trees. *Psychological Review*, **86**, 542–573. doi:10.1037/0033295X.86.6.542

See Also

`eba`, `inclusion.rule`, `strans`.

Examples

```
data(celebrities)           # absolute choice frequencies
A <- list(c(1,10), c(2,10), c(3,10),
          c(4,11), c(5,11), c(6,11),
          c(7,12), c(8,12), c(9,12))  # the structure of aspects
trineq(celebrities, A)        # check trinary inequality for tree A
trineq(celebrities, A)$chkdf  # trinary inequality for each triple
```

---

uscale	*Utility Scale of an EBA Choice Model*

Description

Extract the (normalized) utility scale for an elimination-by-aspects (EBA) model.

Usage

```
uscale(object, norm = "sum", log = FALSE)
```

Arguments

`object`	an object of class eba, typically the result of a call to eba
`norm`	either sum, so the scale values sum to unity (default); or a number from 1 to number of stimuli, so this scale value becomes one; or NULL (no normalization)
`log`	should the log of the utility scale values be returned? Defaults to FALSE.

Details

Each utility scale value is defined as the sum of aspect values (EBA model parameters) that characterize a given stimulus. First these sums are computed for all stimuli, then normalization (if any) is applied. As each type of normalization corresponds to a multiplication by a positive real number, the ratio between scale values remains constant.

Value

The (normalized) utility scale of the stimuli.

See Also

eba, cov.u, wald.test.

Examples

```
data(drugrisk)
A <- list(c(1), c(2,7), c(3,7), c(4,7,8), c(5,7,8), c(6,7,8))
eba1 <- eba(drugrisk[, , group = "male30"], A)  # EBA model

uscale(eba1)                      # sum-to-unity normalization
uscale(eba1, norm=1)              # u(alcohol) := 1
uscale(eba1, norm=5)              # u(heroine) := 1
uscale(eba1, norm=NULL)           # no normalization
uscale(eba1, norm=1, log=TRUE)  # log utility scale, log u(alcohol) := 0
```

---

wald.test	*Testing Linear Hypotheses in Elimination-by-Aspects (EBA) Models*

Description

Tests linear hypotheses of the form $Cp = 0$ in elimination-by-aspects (EBA) models using the Wald test.

Usage

```
wald.test(object, C, u.scale = TRUE)
```

Arguments

`object`	an object of class eba, typically the result of a call to eba
`C`	a matrix of contrasts, specifying the linear hypotheses
`u.scale`	logical, if TRUE the test is performed on the utility scale, if FALSE the test is performed on the EBA parameters directly

Details

The Wald test statistic,

$$W = (Cp)'[Ccov(p)C']^{-1}(Cp),$$

is approximately chi-square distributed with $rk(C)$ degrees of freedom.

`C` is usually of full rank and must have as many columns as there are parameters in `p`.

Value

`C`	the matrix of contrasts, specifying the linear hypotheses
`W`	the Wald test statistic
`df`	the degrees of freedom $(rk(C))$
`pval`	the p-value of the test

See Also

eba, group.test, uscale, cov.u.

Examples

```
data(celebrities)                   # absolute choice frequencies
A <- list(c(1,10), c(2,10), c(3,10),
          c(4,11), c(5,11), c(6,11),
          c(7,12), c(8,12), c(9,12))  # the structure of aspects
eba1 <- eba(celebrities, A)         # fit elimination-by-aspects model

## Test whether JU, CY, and AJF have equal utility scale values
C1 <- rbind(c(0,0,0,1,-1, 0,0,0,0),
            c(0,0,0,1, 0,-1,0,0,0))
wald.test(eba1, C1)

## Test whether the three branch parameters are different
C2 <- rbind(c(0,0,0,0,0,0,0,0,0,1,-1, 0),
            c(0,0,0,0,0,0,0,0,0,1, 0,-1))
wald.test(eba1, C2, u.scale = FALSE)
```

---

winetaste *Wine Tasting Data*

Description

Paired comparison judgments for two wine tasting studies: `ambilight` includes the results of a study on the effect of ambient lighting on the flavor of wine; `redwines` includes judgments on the sensory quality of red wines.

Usage

```
data("winetaste")
```

Format

`ambilight` A data frame containing 230 observations on 10 variables:

preference, fruitiness, spiciness, sweetness Paired comparison of class [`paircomp`](); judgments for one of the 6 ordered pairs of the blue, red, and white lighting conditions.

age subject age

gender factor, subject gender

sensesmell self-rating of sense of smell and taste.

likewine self-rating of general liking of wine.

drinkwine factor, frequency of wine consumption.

redwhite factor, preference for red or white wine.

`redwines` A data frame containing 11 observations on 7 variables:

bitterness, fruitiness, sourness, roundness, preference Paired comparison of class [`paircomp`](); judgments for all 10 pairs from 5 red wines: Primitivo di Manduria, Cotes du Rhone, Bourgogne, Shiraz cuvee, Cabernet Sauvignon.

best factor, Which of the five wines did you like best?

worst factor, Which of the five wines did you like worst?

Details

The `ambilight` data are from Experiment 3 in Oberfeld et al. (2009). The `redwines` data were collected among the members of the Sound Quality Research Unit (SQRU), Department of Acoustics, Aalborg University, Denmark, in 2004. Details of the red wines are available as an attribute of the `preference` variable (see Examples).

References

Oberfeld, D., Hecht, H., Allendorf, U., & Wickelmaier, F. (2009). Ambient lighting modifies the flavor of wine. *Journal of Sensory Studies*, **24**(6), 797–832. doi:10.1111/j.1745459X.2009.00239.x

See Also

eba, eba.order, paircomp.

Examples

```
requireNamespace("psychotools")
data(winetaste)

## No effect of ambient lighting on flavor (Oberfeld et al., 2009)

m <- lapply(ambilight[, c("preference", "fruitiness",
                          "spiciness", "sweetness")],
            function(x) eba.order(summary(x, pcmatrix = TRUE)))
lapply(m, summary)

u <- sapply(m, uscale, norm = 3)
dotchart(
  u, xlim = c(0.5, 2), pch = 16, panel.first = abline(v = 1, col = "gray"),
  main = "Ambient lighting and the flavor of wine",
  xlab = "Utility scale value (Davidson-Beaver model)"
)
ci <- sapply(m, function(x) 1.96 * sqrt(diag(cov.u(x))))
arrows(
  u - ci, c(16:18, 11:13, 6:8, 1:3), u + ci, c(16:18, 11:13, 6:8, 1:3),
.05, 90, 3)
mtext("Oberfeld et al. (2009)", line = 0.5)

## Sensory quality of red wines

psychotools::covariates(redwines$preference)  # details of the wines

m <- lapply(redwines[, c("bitterness", "fruitiness", "sourness",
                         "roundness", "preference")],
            function(x) eba(summary(x, pcmatrix = TRUE)))
lapply(m, summary)

u <- sapply(m, uscale)
dotchart(
  u[order(u[, "preference"]), ], log = "x",
  panel.first = abline(v = 1/5, col = "gray"),
  main = "SQRU red wine tasting",
  xlab = "Utility scale value (BTL model), choice proportion (+)"
)
points(as.vector(
  prop.table(table(redwines$best))[order(u[, "preference"])]
), 1:5, pch = 3)
```

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